A fuzzy guidance law for vertical launch interceptors

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ABSTRACT

A fuzzy logic-based guidance design for longer engagement in time and beneficial interception attitudes is investigated and proposed here. The integrated fuzzy logic-based guidance scheme consisting of vertical, midcourse and terminal guidance phases can intercept the target from diverse aspects. The engagement strategy in the vertical guidance phase includes general vertical launch to handle the case of longer engagement ranges, and vertical launch with back turn (BT) to handle shorter engagement ranges. Numerical experimental results show excellent engagement performance and defensible volumes.

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1. Introduction

The development of efficient guidance strategies for surface-to-air interceptors against very high speed targets is still a critical issue under study and investigation in the defense industry. When a ballistic target re-enters the atmosphere, its speed is extremely high, which makes the remaining time to ground impact relatively short. From the results of previous studies (Imado & Kuroda, 1992; Kuroda & Imado, 1988), a common principle for efficient engagement of ballistic targets has been to build up a near head-on scenario between interceptor and target. The interceptor may ultimately hit the target without resorting to excessive lateral acceleration. In the past, the guidance designs based on the line-of-sight (LOS) angle rate were found effective for interceptor targets with speed far lower than the pursuer. However, new generation targets possess higher speed and larger maneuverability. Thus, classical guidance laws are no longer effective to engage that kind of target (Lin, 1994).

In the literature, several guidance design techniques such as linear quadratic regulator (LQR) (Bryson & Ho, 1975), explicit guidance (Lin & Tsai, 1987), and modified proportional guidance (Newman, 1996) have been proposed for implementing the optimal guidance law. In particular, LQR (Imado & Kuroda, 1992), modified explicit guidance (Lin & Chen, 1998), and neural networks (Song & Tahk, 1998, 1999, 2001) have been applied to deal with the moving targets or anti-tactical ballistic missile (ATBM) guidance design problems. However, solving LQR problems or training neural networks in real-time is known to be practically infeasible in some cases. Song and Tahk (1998) used a neural network to estimate time-to-go with accurate intercept point predictions. After the neural network had been trained according to the approximately optimal guidance law, Song and Tahk (2001) used the trained network to generate suboptimal commands in a real-time implementation. With regard to the issue of attitude control, Oh, Bang, and Park (2008) proposed a design technique that incorporated an adaptive filter for the purpose.

Based on the feasible and easily realizable requirement, a fuzzy guidance design would be an ideal choice; it has a simpler structure than the traditional trajectory shaping guidance and

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Fig. 1. Definitions of the control variables: the angle-of-attack \( \alpha \) and the rolling angle \( \phi \).
does not demand precise modeling knowledge. In addition, the fuzzy inference mechanism is robust to the changes of the environment and is closer to the ideal thought of guidance law designers. Recently, researchers have attempted to apply the fuzzy inference mechanism to missile guidance designs (Lin & Mon, 1999; Mishra, Sarma, & Swamy, 1994). In a previous study (Lin & Chen, 2000; Lin, Hung, Chen, & Chen, 2004), an integrated fuzzy type guidance law for an effective engagement strategy of ballistic targets with a slanted launch interceptor is investigated. However, compared to a slanted launch, a vertical launch interceptor possesses extra advantages that are worthy of greater attention. For example, a vertical launch can engage incoming targets from various aspects. Thus, the launcher need not be rotated so as to aim at the incoming targets before interceptor launch. Furthermore, a vertically launched interceptor can intercept targets that fall into a zone that is unreachable by a slanted launch interceptor.

Two vertical launch guidance laws are proposed in this paper: (i) general vertical launch and (ii) vertical launch with back turn.

Fig. 2. Operational flow of the engagement strategy.

Fig. 3. Engagement scenario for the vertical launch interceptor: (a) without BT and (b) with BT.
(BT), with respect to two kinds of engagement scenarios. Case (i) applies to a longer engagement range with the vertical launch guidance law involving trajectory shaping, midcourse and terminal guidance phases; Case (ii) applies to a shorter engagement range in which the vertical launch with BT is combined with the terminal phase to extend its defensible volume when the slanted launch is incompetent. The fuzzified approach has been proposed for designing the guidance laws for all cases; for reasons that are two-fold: (i) it is well known that fuzzy controllers can use knowledge expressed in the form of linguistic rules without resorting to precise plant models. In the guidance applications, fuzzy logic approaches using if-then rules can simplify the design; and (ii) the fuzzy strategy is robust to changes of environment and is closer to the ideal thought of guidance law designers. To fully verify the proposed approach, simulations are conducted to examine engagement performance of various scenarios.

2. System descriptions

2.1. Interceptor

Consider the 3D translational equations of motion of the guided missile:

\[ \frac{dv_m}{dt} = \frac{T \cos \alpha - D}{m - g \sin \gamma}, \quad v_m(0) = 0, \]

\[ \frac{\dot{y}}{\gamma} = \frac{(L + T \sin \alpha \cos \phi / (m v_m)) - g \cos \gamma / v_m - \gamma(0)}{v_m}, \quad \gamma(0) = \gamma_0, \]

\[ \psi = \frac{(L + T \sin \alpha \cos \phi / (m v_m \cos \gamma)) - \psi(0)}{\gamma(0)}, \quad \psi(0) = \psi_0, \]

\[ \dot{x}_m = v_m \cos \gamma \cos \psi, \quad x_m(0) = x_{m0}, \]

\[ \dot{y}_m = v_m \cos \gamma \sin \psi, \quad y_m(0) = y_{m0}, \]

\[ \dot{h}_m = v_m \sin \gamma, \quad h_m(0) = h_{m0}. \]

Fig. 4. Desired flight trajectory for BT: (a) pitch plane and (b) horizontal plane.

Fig. 5. MFs adopted in the vertical launch phase: (a) $R$ (m), (b) $z$ (m) and (c) $M_z$ (m).
where \( m \) is the missile mass, \( T \) is the thrust, \( \gamma \) is the flight-path angle, \( \psi \) is the azimuth angle, the lift force \( L = (\mu v_{\infty}^2 s_m C_{mL}) / 2 \) and the drag force \( D = (\mu v_{\infty}^2 s_m C_{mD}) / 2 \) with \( C_{mL} = C_{mL0} + \mu s_0 \) and \( C_{mD} = C_{mD0} + \mu s_0 \), \( s_0 \), and \( \mu \) are given as functions of the Mach number, that is, the function of the velocity \( v_{\infty} \) and the altitude \( h_m \). Referring to Fig. 1, the aforementioned angle-of-attack \( \alpha \) is treated as the control variable on the vertical plane and the rolling angle \( \phi \) is treated as the control variable on the horizontal plane guidance law.

### 2. Target

The target vehicle in the reentry phase to be located over a flat, non-rotating earth with gravity is considered. The ballistic target model in the radar coordinates centered at radar site is assumed to be:

\[
\begin{align*}
\dot{v}_x &= -\frac{\rho v_{\infty}^2}{2\rho} g \cos \gamma_1 \sin \gamma_2 + a_{tx}, \quad v_{tx}(0) = v_{tx0}, \\
\dot{v}_y &= -\frac{\rho v_{\infty}^2}{2\rho} g \cos \gamma_1 \cos \gamma_2 + a_{ty}, \quad v_{ty}(0) = v_{ty0}, \\
\dot{v}_h &= -\frac{\rho v_{\infty}^2}{2\rho} g \sin \gamma_1 - g + a_{th}, \quad v_{th}(0) = v_{th0},
\end{align*}
\]

where \( v_{tx}, v_{ty}, \) and \( v_{th} \) denote the velocity components of \( v_t \) in the \( X, Y \) and \( H \) axes, respectively; \( a_{tx}(t), a_{ty}(t) \) and \( a_{th}(t) \) are the uncertain accelerations due to maneuvering; the flight path angles \( \gamma_1 \) and \( \gamma_2 \) and the ballistic coefficient \( \beta \) are given as:

\[
\begin{align*}
\gamma_1(t) &= \tan^{-1} \left( -\frac{v_{th}}{\sqrt{v_{tx}^2 + v_{ty}^2}} \right), \\
\gamma_2(t) &= \tan^{-1} \left( \frac{v_{tx}}{v_{ty}} \right), \\
\beta &= \frac{W}{s_0 C_{D00}},
\end{align*}
\]

where \( s_0, W, \) and \( C_{D00} \) represent the reference area, weight and zero-lift drag coefficient of the ballistic target, respectively.

### 3. Engagement strategy

The proposed engagement strategy is divided into two schemes, depending on the interceptor–target relative range and height. For the case of general vertical launch guidance (GVLG), the interceptor–target range is long enough for the guidance system to complete four guidance phases: vertical, midcourse, shaping and terminal guidance phases. For the case of vertical launch with back-turn guidance (VLBG), the target has already come close to the interceptor’s launcher. It is usually lacking sufficient distance to build up speed and pose itself for effective engagement. Two guidance phases, that is the vertical and terminal guidance phases, are proposed to deal with the situation. The interceptor body must make a BT after launching for the purpose of engaging. The operational flow of the engagement

**Table 1**

<table>
<thead>
<tr>
<th>( M_z )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>( M )</td>
</tr>
<tr>
<td>( Z_t )</td>
<td>( L )</td>
</tr>
<tr>
<td>( M )</td>
<td>( L )</td>
</tr>
<tr>
<td>( H )</td>
<td>( M )</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>( \sigma_{\phi} )</th>
<th>( \gamma )</th>
<th>( \gamma - \gamma_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN</td>
<td>LN</td>
<td>LN</td>
</tr>
<tr>
<td>LN</td>
<td>LN</td>
<td>SN</td>
</tr>
<tr>
<td>ZE</td>
<td>SN</td>
<td>SN</td>
</tr>
<tr>
<td>SP</td>
<td>ZE</td>
<td>ZE</td>
</tr>
<tr>
<td>LP</td>
<td>LP</td>
<td>LP</td>
</tr>
</tbody>
</table>

**Fig. 6.** Illustrations of the designed altitude for the vertical guidance law.

**Fig. 7.** MFs for the BT in the vertical launch mode: (a) \( \gamma - \gamma_d \) (deg), (b) \( \dot{\gamma} \) (deg/s) and (c) \( \sigma_{\phi} \) (deg).
strategy is illustrated in Fig. 2, where the notation Ω denotes the defensible volume covered by GVLG.

The fuzzy logic based guidance laws in midcourse, shaping, and terminal phases have been well developed in the literature. Here, the focus is particularly on the development of the vertical guidance law and its interaction with midcourse and terminal guidance.

3.1. Vertical guidance strategy

Suppose that the aerodynamically controlled interceptor is asymmetric in its body. Then, there are two actions to be implemented during the vertical guidance phase. First, the interceptor body has to roll to an orientation toward the target; this simplifies the maneuver and reduces the required lateral acceleration in the horizontal plane. Second, the interceptor body is forced to incline and commence the course of engagement while it reaches a favorable altitude.

During the initial stage of the vertical launch, the interceptor is not pointed at the target. Therefore, the accurate interceptor speed with respect to the target is hard to calculate. To approximate an orientation toward the target, consider the rolling control command:

$$\phi = \tan^{-1} \left( \frac{R_{m0} h_{m0}}{R_{m0}} \right),$$

where $$R_{m0} = x_{m0} + v_{m0}\Delta t_{m0} - x_{m0}$$ and $$R_{m0} = y_{m0} + v_{m0}\Delta t_{m0} - y_{m0}$$ with $$x_{m0}(v_{m0})$$ and $$y_{m0}(v_{m0})$$ being the initial position (velocity) of the target in the X and Y axes, respectively; $$\Delta t_{m0}$$ is the average time of the interceptor in fulfilling the required rotation.

3.2. GVLG

The next step is to drive the interceptor to trace a desired flight path toward the target: see Fig. 3(a). This is fulfilled by exerting a negative pitch acceleration to change the altitude of the missile body. The strategy is to choose the pitch flight path error angle $$\gamma - \gamma_d$$, with $$\gamma_d$$ denoting the desired flight path angle (referred to as the ideal launch angle that yields the largest defensible volume), and its change rate as the antecedent variables of a fuzzy inference system $$f_d(-, -)$$ to infer the guidance command $$\Delta f_d$$ (discussed in Section 4.1).

The desired pitch angle at this stage is defined by

$$\gamma_d = \tan^{-1} \left( \frac{h_t - h_{m0}}{x_t - x_{m0}} \right),$$

where $$(x_{m0}, h_{m0})$$ are the interceptor’s instantaneous coordinates in the pitch plane when the inclination starts, and

$$x_t = x_t + v_{tx}t, \quad y_t = h_t + v_{ty}t,$$

with $$t$$ being the time-to-go to the preset lock-on point:

$$t = \frac{\text{PIP}_t}{\text{PIP}_s},$$

in which the predicted lock-on range $\text{PIP}_s$ is given by

$$\text{PIP}_s = \frac{\sqrt{R_{mxys}^2 + R_{mty}^2}}{v_{mxys} - v_{mty}} + R_{lock},$$

where $$R_{mxys} = \sqrt{R_{mx}^2 + R_{mty}^2}$$, $$v_{mxys} = \sqrt{v_{mx}^2 + v_{mty}^2}$$, and $$R_{lock}$$ is the seeker lock-on range, which is assumed to be constant.

3.3. VLBG

When the target has already come close to the top of the launcher, raising the interceptor body should be completed: see Fig. 3(b). The desired interceptor flight trajectory for the vertical plane is illustrated in Fig. 4(a) in which the BT point is denoted by M with the turning angle $$\gamma_t = \gamma_1$$, where $$\gamma_1$$ is the target’s flight path angle in the pitch plane and $$M_t$$ is the desired height of the turning point. Given $$\gamma_1$$, PIP denotes the predicted interception point and let

$$\text{PIP}_d = \text{PIP}_s - M_t.$$

![Fig. 8. MFS for $M_t$ in the vertical launch case with BT: (a) $R$ (m), (b) $z_t$ (m) and (c) $M_t$ (m).](image-url)
Therefore

\[ \mathbf{b}_M = \frac{\mathbf{P}_{IP_d}}{\tan \gamma_1} - \mathbf{P}_{IP_x}, \]

where \( \mathbf{P}_{IP_x} \) and \( \mathbf{P}_{IP_z} \) are, respectively, the distances between the launcher and the target in the X and H axes with respect to \( \mathbf{P}_{IP} \). Let the range \( \mathbf{L}_M \) to be:

\[ \mathbf{L}_M = b_M \tan \gamma_1, \]

Then, the desired flight path from \( M \) to the counter parallel flight path is

\[ F_M = \mathbf{L}_M \cos \gamma_1. \]

The coordinates of the point \( F \) in the pitch plane are given by \((F_M \sin \gamma_1, F_M \cos \gamma_1 + M_z)\). The entire operation consists of two parts: Phase I—when the interceptor reaches the specified turning point \( M \), it is ready to make a turn with \( \gamma_d \) set to be \( \gamma_1 + 0.5\pi \); Phase II—\( \gamma_d \) is changed to \( \gamma_1 \) when the interceptor attains the midway point, denoted by \( N = (N_x, N_y, N_h) \), of the flight path \( F_M \). Similarly, the midway point of \( \mathbf{L}_M \) is denoted by \( Q = (Q_x, Q_y, Q_h) \).

For the horizontal plane as illustrated in Fig. 4(b), when the interceptor attains the BT point, it is required to make a turn and the desired flight path angle is set to be \( \gamma_2 + 0.5\pi \) before it attains \( \mathbf{L}_M \) with \( \gamma_2 \), denoting the target’s flight path angle in the horizontal plane.

To achieve an exact orientation in the pitch plane, consider the relative motion between the interceptor and target.

Clearly,

\[ R_{msh} = -v_m \cos \sigma_v, \]

\[ R_{msh}\theta_v = -v_m \sin \sigma_v. \]

Differentiating Eq. (19) with respect to time and combining these equations yields

\[ \ddot{\theta}_v = \left( \frac{\dot{v}_m}{v_m} \cdot \frac{2R_{msh}}{R_{msh}} \right) \dddot{v}_m + \frac{R_{msh}}{R_{msh}} \dddot{v}_m. \]

Without loss of generality, assume \( \dot{v}_m/v_m \approx 0 \). Then

\[ \ddot{\theta}_v = \frac{1}{t_{\text{op}}^2} (2\dot{\theta}_v - \dot{\gamma}_d). \]

where \( t_{\text{op}} = -R_{msh}/R_{msh}. \)

Consider the minimization of the following cost function:

\[ J = \int_0^{t_f} \dot{\gamma}^2(t) \, dt, \]

subject to Eq. (21) and \( \theta(t_f) = \gamma_d \), \( \dot{\theta}(t_f) = 0 \). The optimal solution can be found which is given by

\[ \dot{\gamma}_d = \frac{2}{t_{\text{op}}} (\theta_v - \gamma_d) + 4\dot{\theta}_v. \]

On the basis of this result, the guidance law is generalized in the following form:

\[ a_{mp} = v_m \gamma_d - N_1 v_m \left( \gamma_d - \theta_v \right) + N_2 v_m \dot{\theta}_v, \]

where \( N_1 \) acts as the guidance gain with respect to the heading error \( \gamma_d - \theta_v \), whereas \( N_2 \) plays a similar role as the conventional proportional navigation guidance gain, and

Phase I: \( \gamma_d = \gamma_1 + 0.5\pi \), \( t_{\text{op}} = -R_{msh}/R_{msh}. \)

\[ R_{msh} = \sqrt{(N_x - x_m)^2 + (N_h - h_m)^2}. \]

Phase II: \( \gamma_d = \gamma_1 \), \( t_{\text{op}} = -R_{msh}/R_{msh}. \)

\[ R_{msh} = \sqrt{(Q_x - x_m)^2 + (Q_h - h_m)^2}. \]

Fig. 9. Defensible volumes for the vertical launch case in: (a) fourth quadrant and (b) 3D space.
In order to allow the interceptor to reach a near head-on geometry before entering the terminal phase, the midcourse guidance law and the range to the preset lock-on point is designed as:

\[ R_p = R + R_{lock} \]  

(23)

where \( R = \sqrt{R_{mx}^2 + R_{my}^2 + R_{mz}^2} \). The estimated time-to-go to the preset lock-on point is

\[ t_{gop} = \frac{R_p}{R_{mx} v_{nx} + R_{my} v_{ny} + R_{mz} v_{nz}} \]  

(24)

where \( v_{nx} = v_{mx} \cos \gamma \cos \psi \), \( v_{ny} = v_{my} \cos \gamma \sin \psi \), and \( v_{nz} = v_{m} \sin \gamma \). The desired flight-path angles on both planes are specified as:

\[ \gamma_d = \tan^{-1} \left( \frac{h_f - h_m}{x_f - x_m} \right) \]  

\[ \psi_d = \tan^{-1} \left( \frac{y_f - y_m}{x_f - x_m} \right) \]  

(25)

where \( x_f = x_t + v_{nx} t_{gop} \), \( y_f = y_t + v_{ny} t_{gop} \) and \( h_f = h_t + v_{nz} t_{gop} \).

The predicted LOS angle with respect to the lock-on point is given by

\[ \hat{\gamma}_v = \tan^{-1} \left( \frac{h_v - h_m}{x_v - x_m} \right) \]  

\[ \hat{\psi}_v = \tan^{-1} \left( \frac{y_v - y_m}{x_v - x_m} \right) \]  

(26)

Fig. 10. Related response profiles correspond to the target with the reentry coordinates (29 000, 28 500, 44 300) (m): (a) relative distance between interceptor and target in the X-H plane and (b) X-Y plane; (c)–(d) velocity error angle and heading error angle of the vertical plane; (e)–(f) velocity error angle and heading error angle of the horizontal plane.
Table 4
Simulation results for the vertical launch without BT.

<table>
<thead>
<tr>
<th>Item</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
<th>Point D</th>
<th>Point E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final velocity (m/s)</td>
<td>644</td>
<td>681</td>
<td>566</td>
<td>321</td>
<td>399</td>
</tr>
<tr>
<td>Final position (m)</td>
<td>(3507,3507,20,493)</td>
<td>(5524,5524,20,842)</td>
<td>(7900,7000,19,808)</td>
<td>(1312,612,17,399)</td>
<td>(1697,1077,18,637)</td>
</tr>
<tr>
<td>Final time (s)</td>
<td>24.8</td>
<td>23.4</td>
<td>26.4</td>
<td>24.8</td>
<td>21</td>
</tr>
<tr>
<td>PIP (m)</td>
<td>134,520</td>
<td>140,730</td>
<td>134,640</td>
<td>134,520</td>
<td>27,325</td>
</tr>
<tr>
<td>MD (m)</td>
<td>5.2</td>
<td>9.4</td>
<td>7.3</td>
<td>9.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

where \( x_t = x_t + v_{x_d} t, y_t = y_t + v_{y_d} t \) and \( h_t = h_t + v_{h_d} t \) with

\[
\dot{t} = \frac{R_{lock}}{R_p} = \frac{R_{lock} R}{R_{max} v_{tx} + R_{my} v_{ty} + R_{mn} v_{tm}}.
\]  

The predicted velocity error angles are defined by

\[
\hat{\delta}_v = \gamma_d - \gamma, \quad \hat{\delta}_h = \psi_d - \psi,
\]

and the corresponding heading error angles are defined as:

\[
\sigma_v = \gamma - \hat{\delta}_v, \quad \sigma_h = \psi - \hat{\delta}_h.
\]

4. Fuzzy logic guidance law design

4.1. Fuzzy guidance law—vertical launch without BT

Vertical roll \( M_2 = f_s(Z, R) \): The linguistic variables for the guidance law are \( z_t \) and \( R \), and the output variable is the desired height \( M_2 \). Each linguistic variable is assumed to be characterized by three linguistic sets, see Fig. 5. The illustration of the ideal arrangement is displayed in Fig. 6, which is realized by the nine rules in Table 1. Finer guidance decisions are generally obtainable by expanding the rule base.

Attitude change \( \bar{z}_b = f_s(\gamma - \gamma_d, \dot{\gamma}) \): The input variables are the flight path error angle \( \gamma - \gamma_d \) and its rate of change \( \dot{\gamma} \). The output variable is \( \bar{z}_b \). Each linguistic variable takes five linguistic sets: see Fig. 7. The fuzzy rule base is given in Table 2. The design criteria can be summarized as follows:

**Group 1:** Both \( \gamma - \gamma_d \) and \( \dot{\gamma} \) are very small or close to zero. This means that the current LOS angle is close to \( \gamma_d \). Thus, the amount of control action is small and intended to slightly correct the deviation.

**Group 2:** \( \gamma - \gamma_d \) and \( \dot{\gamma} \) are positive. The interceptor’s flight-path angle is larger than \( \gamma_d \) and it is flying downward. Control action is thus designed to lift the interceptor.

**Group 3:** \( \gamma - \gamma_d \) is negative and \( \dot{\gamma} \) is positive. This means that the interceptor’s flight-path angle is less than \( \gamma_d \) and it is flying downward. Control action is thus designed to further lower the interceptor’s altitude to speed up the convergence of \( \gamma - \gamma_d \).

**Groups 4 and 5:** These rules are opposite to Groups 3 and 2, respectively.

The design ideas of the subsequent fuzzy midcourse and terminal guidance laws follow the ideas proposed by Lin and Chen (2000), Lin et al. (2004).

Fuzzy midcourse guidance law (FMGL) \( \bar{z}_m = f_m(\sigma_v, \delta_v) \): Each of the linguistic variables is assumed to be characterized by five fuzzy sets. The velocity error angle \( \delta_v \) is considered more important than the heading error angle \( \sigma_v \); the former is thus weighted heavier.

Fuzzy shaping guidance law (FSGL) \( \bar{z}_s = f_s(\sigma_v, \delta_v) \): Since the position error dominates MDs, a feasible approach keeps applying the midcourse guidance law but linearly reduces \( \delta_v \) and \( R_{lock} \) to zero so that the inference command is gradually dominated by the position error. Other operations remain invariant as those in the midcourse phase.

Fuzzy terminal guidance law (FTGL) \( \bar{z}_t = f_t(\sigma_v, \delta_v) \): As the interceptor enters the terminal phase, the velocity error in determining the guidance command becomes a minor concern. It would be appropriate to apply the position error \( \sigma_v \) alone while facilitating the terminal guidance design. Thus a fuzzy PD-like controller is proposed. A rule table that is analogous to the standard PD fuzzy rule base can accordingly be applied.

The idea above can be directly applied to the horizontal guidance design, except that the influence due to gravity can be ignored. In addition, the control variable \( \alpha \) becomes \( \phi \), \( \gamma \) becomes the azimuth angle \( \psi \), and \( \theta_n \) becomes the inertial LOS angle \( \theta_h \) in the horizontal plane.

4.2. Fuzzy guidance law—vertical launch with BT

Three actions have to be implemented during the vertical guidance phase: vertical roll, BT and twist. The first fuzzy rule base is to determine an appropriate back turning height \( M_s \) with the input variables \( z_t \) and \( R \). The corresponding MFs are defined in Fig. 8. The guidance rules have been depicted in Table 1, which is designed with reference to the standard PD fuzzy control rules.
The guidance command for changing the interceptor’s attitude is generated by Eq. (22), using the heading error angle $\gamma_d - \theta_v$ and $\theta_v$.

The guidance law switches to the terminal guidance phase after BT is completed. That is, the interceptor has already attained the extended line of abF in Fig. 4(a) with the flight path angle $\gamma \approx \gamma_1$. For brevity, the details are omitted.

The inferred guidance commands defined above are defuzzified before sending to the autopilot. The crisp control action is calculated using the center average defuzzifier to ensure the smoothness of the resulting guidance commands.

5. Results and analyses

Parameter settings for the interceptor are given in Table 3. A Poisson evasive model was adopted to emulate the random target maneuver with the magnitudes of $a_y(t)$ and $a_m(t)$ being 5 g. Time constant of the first-order shaping filter realized for the model was 1 s. The criterion for target engagement is characterized by

\[
J = \frac{1}{n} \sum_{i=1}^{n} |MD_i| \leq MD_{\text{max}}
\]

s.t.

\[
a_{\text{total}}(t) = \sqrt{a_{\text{mp}}^2(t) + a_{\text{mx}}^2(t)} \leq a_{\text{c,max}}, \quad 0 \leq t \leq t_f,
\]

where $n$ is the number of total Monte Carlo runs (in this research $n = 10$ was set), $MD_{\text{max}} = 10$ m is allowable maximal miss distance, $t_f$ is the fight time, $a_{\text{c,max}} = 35$ g is the allowable maximal lateral acceleration of the missile body with

\[
a_{\text{mp}} = \frac{1}{m} L \cos \phi - g \cos \gamma, \quad a_{\text{mx}} = \frac{1}{m} L \sin \phi.
\]

The MD for each scenario depicted later is the mean value of a total of 10 Monte Carlo runs.

![Fig. 12. Engagement trajectories corresponding to the target with the reentry coordinates (16 600, 16 730, 38 500) (m); (a) relative distance between interceptor and target in the X-H plane and (b) X-Y plane; (c)-(d) velocity error angle and heading error angle of the vertical plane; (e) missile velocity.](image-url)
Table 5
Simulation results of the vertical launch with BT.

<table>
<thead>
<tr>
<th>Item</th>
<th>Point F</th>
<th>Point G</th>
<th>Point H</th>
<th>Point I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final velocity (m/s)</td>
<td>513</td>
<td>500</td>
<td>381</td>
<td>344</td>
</tr>
<tr>
<td>Final position (m)</td>
<td>(45,55,19,520)</td>
<td>(13,13,19,392)</td>
<td>(60,300,18,131)</td>
<td>(150,130,16,200)</td>
</tr>
<tr>
<td>Final time (s)</td>
<td>24</td>
<td>24.3</td>
<td>25.5</td>
<td>27</td>
</tr>
<tr>
<td>Minimum distance (m)</td>
<td>64,955</td>
<td>64,955</td>
<td>57,509</td>
<td>49,860</td>
</tr>
<tr>
<td>Maximum distance (m)</td>
<td>8.6</td>
<td>6.3</td>
<td>7.2</td>
<td>9.9</td>
</tr>
</tbody>
</table>

5.1. Vertical launch without BT

The guided missile was required to intercept the target with high terminal speed. The resulting defensible volume is shown in Fig. 9, which shrinks with the increasing altitude. This is due to the natural deterioration of maneuverable efficiency for aerodynamically controlled missiles at higher altitudes. Obviously, this is consistent with the previous study for the slanted launch case, which showed that the defensible volume would be larger when the launcher’s slanted angle is about 45° (Lin & Chen, 2000).

Related tracking responses and control histories for the target with the reentry coordinates (29,000, 28,500, 44,300) (m) in Fig. 9(a) are shown in Fig. 10. It can be seen from the heading error response that the interceptor reaches a near-head-on geometry before it switches to the terminal guidance phase. Table 4 summarizes the final results for other representative scenarios. The cylinder indicated by dashed lines within the defensible volume of Fig. 9(b) is the indefensible zone where the interceptor cannot build sufficient speed for effective engagement due to insufficient relative ranges. For these scenarios, the strategy of a vertical launch with BT is resorted.

5.2. Vertical launch with BT

We demonstrate results for the target with the reentry coordinates (16,600, 16,730, 38,500) (m). When the interceptor attained the turning point as shown in Fig. 4, the remaining relative range between the intercepter and the target was less than 15 km; this was an insufficient distance for the interceptor to build up its speed for an effective interception. The improvement of defense capability by applying the proposed guidance law is illustrated in Fig. 11. The engagement trajectories and major state responses are displayed in Fig. 12. Table 5 summarizes the final results of other representative scenarios.

6. Conclusions

We have developed a novel integrated fuzzy logic-based guidance scheme so as to guide a vertical launch interceptor to intercept high speed reentry targets. The engagement strategy includes general vertical launch and vertical launch with back turn, which are both shown to be appropriate in dealing with cases of longer or shorter engagement ranges. The guidance law at each engagement phase is realized by a fuzzy logic inference system to incorporate fine guidance commands. Extensive numerical verifications show satisfactory engagement performance of the proposed design.

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References


